3101. No calculus is required here. Consider the signed areas graphically. You can visualise the problem with $y = (x - k)^2$.

3102. Find the points of inflection by setting the second derivative to zero (and checking it changes sign). Then show that $y\sqrt{e} = 2 - x$ is tangent to the curve at this point.

3103. Just count up V, E, F for each!

3104. In both parts, work out the number of successful branches of the (imagined) tree diagram and set up calculations of the form

$$p = {}^n\mathrm{C}_r \times p_1 \times p_2 \times p_3 \times p_4.$$

In both, there are four different probabilities p_i.

3105. Draw a clear sketch, and use calculus.

3106. Write $y = x^3 - 8x$ in the form $y = a\big((bk)^3 - bk\big)$, where a and b generate stretches in the y and x directions respectively.

3107. Consider the extreme cases, in which all the load is taken by the central support, or by both the outer supports.

3108. Consider fully the nature of the point $(q, 0)$.

3109. One is true and two are false.

3110. Rewrite the integral equation as a differential equation (differentiate both sides), and solve.

3111. Substitute the second equation into the former, to produce a quadratic equation in x with coefficients in terms of k. Set up and solve $\Delta \geq 0$.

3112. Translate the If statement into algebra, starting $\mathrm{g}(-x) = ...$, and differentiate it.

3113. Consider $\bar{y}$ as the mean of the length of vertical lines from the x axis to the graph $y = \mathrm{f}(x)$.

3114. Let θ be the angle of projection. Find the maximal height h_{max} of the projectile in terms of u, g and θ. Then set up the inequality $h_{\text{max}} \geq \frac{u^2}{2g}$ and solve.

3115. (a) For the domain, work out if the denominator has any roots.
(b) For the range, note that $y \to 0$ as $x \to \pm\infty$. Hence, the range must have a bound at one or more SPs.

3116. Consider the transformation which takes the first parabola onto the second. Apply this algebraically to $y = px^2 + qx + r$.

3117. The fact that the gradient is non-zero rules out degree 0. Then the fact that the gradient has the same value for two distinct inputs rules out degree 2.

3118. The expression is a cubic in $(a^3 + 1)$.

3119. (a) Show that the curve is increasing everywhere.
(b) Show that the second derivative is zero and changes sign.
(c) Draw a sketch of the first couple of iterations of the N-R method.

3120. Consider the rotational symmetry of the graph.

3121. (a) Expand using the compound-angle formula and then use two double-angle formulae.
(b) Differentiate and then use the identity from (a) to factorise.

3122. Consider the fact that the equation $\mathrm{f}''(x) = 0$ must have no real roots.

3123. (a) Establish that the string forms two equilateral triangles at each side of the square, and then resolve horizontally at a pulling point.
(b) Add two tensions together (as vectors) to give a resultant force.

3124. (a) Make the first instance of x the subject.
(b) Differentiate and consider the value of $|\,\mathrm{g}'(x)|$ in the vicinity of $x = 0.1$.

3125. Find the exact squared side length using e.g. the latter two vertices. Then find the interior angle using the cosine rule.

3126. Consider the behaviour in and outside the domain $(-1, 1)$.

3127. Use the derivative $\frac{d}{dx}\operatorname{cosec} x = -\cot x \operatorname{cosec} x$ and the reverse chain rule.

3128. Call the bottom left point $(0, 0)$, the top right point (m, n), and the other lattice point on the diagonal (a, b). Equate gradients.

3129. (a) Find a formula for $\sum y_i$, by splitting up the sum. For this, you need a formula for $\sum x_i^2$. You can write this in terms of the s_x^2, n and $\bar{x}$, using the formula for the sample variance.
(b) Try to do the same thing as in (a). Show that one of the required quantities is unknown.

3130. Find the coordinates and classification of the SPs. Then just join the dots!

3131. Rearrange the inequality to $x^2 \geq \frac{1}{x}$. Set up and solve the boundary equation. Then sketch $y = x^2$ and $y = \frac{1}{x}$ on the same set of axes.

3132. In each, the counterexample should consist of a pair of functions f and g.

3133. (a) Differentiate, find the normal gradient, and substitute into $y - y_1 = m(x - x_1)$.
(b) Find the equations of the normals which pass through $(0, 2)$ and $(2, -0.5)$. Then, find the coordinates of the closest points, and, by Pythagoras, the (squared) distances.

3134. (a) Split an (isosceles) sector of the polygon into two right-angled triangles.
(b) Rearrange the result of (a) to make $n \sin \frac{180°}{n}$ the subject. Consider the fact that, when an n-gon has many sides, it is nearly circular.

3135. Consider a transformation of $y = \frac{1}{x^6}$.

3136. Show that the denominator is never zero, using Δ.

3137. The trivial case is where every pupil in the group got exactly the same scores X and Y. Setting this aside, consider the relationship between X and Y as the equation of a line, in which M is constant.

3138. Find the coordinates of the image of the vertex.

3139. The average value of a function is calculated by integrating over the domain, and then dividing by the width of the domain.

3140. (a) Plug the numbers in.
(b) Consider the relative magnitudes of x^4 and x^2 for $x \in (0, 1)$.

3141. Express $\mathbb{P}(X = 0)$ and $\mathbb{P}(X = 0, 1, 2)$ in terms of $q = 1 - p$. Set up an equation and solve.

3142. Use substitution and simplify with the binomial expansion. Then use a polynomial solver.

3143. Substitute $y = f(x) + c$ into the DE.

3144. (a) Let p to be the probability that any carpenter in the population works left-handed.
(b) Using $X \sim B(n, 0.1)$, solve $\mathbb{P}(X = 0) = 0.01$.
(c) Find the c.r. and write down the probability, assuming H_0, that the test statistic is in it. You're looking for a probability a bit smaller than the significance level.

3145. Use a Pythagorean trig identity.

3146. The second derivative being zero is necessary, but not sufficient, for a point of inflection.

3147. (a) Expand binomially, and substitute the correct x to produce $\sqrt{1.08}$.
(b) Consider the fact that 1.08 is three times 0.36, which is a perfect square.

3148. Write $f(x) = (x - \alpha)^2 g(x)$, where $g(x)$ is some polynomial. Then differentiate.

3149. (a) Find $\cos\theta$ in terms of n. Then use the first Pythagorean identity: $\sin\theta = \sqrt{1 - \cos^2\theta}$.
(b) Substitute the value for $\sin\theta$ from (a), square both sides and simplify.
(c) Multiply out to get a quartic in n. Solve this using your calculator.

3150. Rearrange each parametric equation to make the trig ratio the subject. Then square the equations and use the second Pythagorean trig identity.

3151. There are $4! = 24$ orders of $\{x_1, x_2, x_3, x_4\}$. The given fact $x_1 < x_4$ cuts the possibility space by half, as $x_1 < x_4$ and $x_4 < x_1$ are equally likely.

3152. Find the first and third derivatives of $\sin x$, and substitute into the LHS.

3153. Set up (x, y) axes, with the origin in the centre of the hexagon. Then find the equation of the sloped edge in the first quadrant. Solve simultaneously with $y = x$.

3154. Find the pairwise points of intersection, and use $A = \frac{1}{2}bh$.

3155. Differentiate both sides (the LHS implicitly) with respect to x. Rearrange to find $\frac{dy}{dx}$. Work out the equation of the normal at (p, q), and show that it passes through the origin.

3156. Express the equation as a quadratic in 3^{4x}.

3157. Differentiate by the product rule.

3158. In both (a) and (c), you know the common ratio. In (b), you don't; the question is whether you can still calculate the 99th term.

3159. You need to show that applying a mod function to $f(x)$ doesn't do anything. In other words, you need to show that $f(x)$ is never negative.

3160. There are two cases to consider, depending on the sign of $(\sec\theta+\tan\theta)$. Differentiate $\ln(\sec\theta+\tan\theta)$ and then $\ln(-\sec\theta-\tan\theta)$, by the chain rule.

3161. This is false. Consider repeated roots.

3162. (a) Firstly, consider the behaviour of $x=\sin t$ and $y=\tan t$ separately, particularly the range over the domain $[0,2\pi)$. Use the fact that $\sin t$ and $\tan t$ have the same rate of change at $t=0$.
(b) Consider the acceleration and therefore force in the y direction.

3163. This is about the multiplicity of the roots of the equation $\text{f}_1(x)=\text{f}_2(x)$.

3164. Work out the average (mean) value of $\text{f}(x)$ on the domain $[a,b]$ from the definite integral. Then use the fact that f is linear.

3165. Since the normal distribution is symmetrical, you can consider only $Y\geq\mu$. So, calculate

$$\mathbb{P}(Y>\mu+2\sigma \mid Y>\mu+\sigma),$$

using the conditional probability formula. This is easiest using $Z\sim N(0,1)$.

3166. Use a double-angle formula, and then factorise.

3167. Draw a sketch of the two boundary cases, in which the second circle is tangent to the first and
① inside it, or
② outside it.

3168. (a) Draw a diagram, with A and B adjacent to each other. Then use the "angle at the centre" theorem.
(b) Apply the same method as in (a) to the various different locations of B relative to A.

3169. Let the functions be

$$\text{f}_1(x)=\frac{\text{g}_1(x)}{\text{h}_1(x)},\quad \text{f}_2(x)=\frac{\text{g}_2(x)}{\text{h}_2(x)},$$

where $\text{g}_1,\text{g}_2,\text{h}_1,\text{h}_2$ are polynomial functions. Compose the functions as $\text{f}_1\text{f}_2(x)$. Simplify using (a generalised version of) the technique for inlaid fractions: multiply top/bottom of the big fraction by the denominator(s) of the inlaid fractions.

3170. (a) Differentiate.
(b) Set up an equation using 3D Pythagoras and the vector from (a).

3171. Let the triangle have sides $(a,a,2h)$. Then set up simultaneous equations for the perimeter and area. Solve the resulting cubic using a calculator.

3172. Consider the multiplicity of the roots.

3173. (a) Find, in terms of the θ, the area of the segment above $y=\sin\theta$. Equate this to half of the area of the semicircle.
(b) The iteration is $\theta_{n+1}=\frac{1}{4}(\pi-2\sin 2\theta_n)$. Run this with $\theta_0=0$.
(c) Define $\text{f}(\theta)=4\theta+2\sin 2\theta-\pi$. Evaluate f at bounds above and below your value from (b). Show that there is a sign change.

3174. This is a non-separable DE. So, you can't apply the standard method of separation of variables. Set up a generic line as $y=mx+c$. Sub into the DE, and equate coefficients.

3175. (a) Multiply the probabilities that each guest draws out an odd digit.
(b) A set is not ordered, so n doesn't have to be pulled first. Use a combinatorial approach, i.e. $p=\frac{\text{successful}}{\text{total}}$.

3176. By symmetry, the distance between the rollers must remain constant. This is half the length of the block. Consider what placement of the rollers beneath the block will lead to tipping.

3177. Let $u=5x-1$. Enact the substitution, and then multiply out the brackets and integrate term by term.

3178. Either use the discriminant to identify a double root (easier), or differentiate.

3179. Solve the boundary equation $\sin(x+y)=1/2$ for $x+y$, and consider your solution as a set of lines in the (x,y) plane.

3180. Find the height with inclination 2°, by setting up a horizontal *suvat* for t and then a vertical *suvat* for h. Repeat the calculation for 3°.

3181. Solve to find axis intercepts and stationary points. Then consider the behaviour as $x\to\pm\infty$.

3182. Find the coordinates of the intersection, and then set up a definite integral. For each, since asked to "find", you can use the relevant facility on a calculator.

3183. Find the sum of the integers from 1 to 1000 which are multiples of 17. This is the sum of an AP.

3184. Sub the parametric equations into the Cartesian equation of the unit circle, and solve for t.

3185. Differentiate the RHS by the chain rule, using $(\ln x)'$ to represent the derivative of $\ln x$.

3186. Work out the Cartesian equation of the first line segment in the form $ax + by = c$. Also, work out the coordinates of the endpoints of the second line segment. Then test the endpoints in $ax + by$, and compare the results to c.

3187. Use the product rule twice (implicitly) on xy. On the way, don't rearrange to find $\frac{dy}{dx}$.

3188. It's the perpendicular bisector of $(0, 0)$ and (p, q).

3189. Give a counterexample to the first statement, and prove the second using the chain rule.

3190. Factorise the bottom, and divide top and bottom by a common factor.

3191. Consider the constant of integration.

3192. The graphs are reflections of each other in $y = x$. Find the intersections of e.g. the first curve with $y = x$, then set up an integral for the area enclosed by that curve and $y = x$.

3193. The function is a positive sextic. So, it must have a global minimum. Find this value using calculus, or by completing the square on a quadratic in x^3.

3194. Subtract the first from the second, and also the first from the third.

3195. Since the population is large, each individual has the same (or more precisely negligibly different) probability of being above the 90th percentile.

3196. Assume, for a contradiction, that a^2b^3 is a perfect square. Write this algebraically as an equation, and then ask: "How many prime factors of b are there on each side of the equation?"

3197. A fixed point satisfies $\text{f}(x) = x$. Graphically, such points are intersections of the line $y = \text{f}(x)$ and the line $y = x$. Consider the points on $y = \text{f}(x)$ for which the distance between $y = \text{f}(x)$ and $y = x$ is a local maximum.

3198. Rearrange for x in terms of u, substitute in and expand.

3199. Find the vertices of the quadrilateral by solving various simultaneous equations. To find its area, encase it in a rectangle with sides parallel to the x and y axes. Subtract the areas of the four triangles so formed from the area of the rectangle.

3200. This question uses complicated language, but there is little algebra (a simple integral) to do once you've understood it. The key fact to use is that the angle α between a line $y = mx + c$ and the x axis satisfies $\tan \alpha = m$.

— End of 32nd Hundred —